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(May 12, 2000)

Structure formation with cold dark matter (CDM) predicts halos with a central density cusp, which are observationally disfavored. If CDM particles have an annihilation cross section $\sigma v \sim 10^{-28}(m/\,\mathrm{GeV})\,\mathrm{cm}^2$, then annihilations in the dense, central parts of halos will soften the cusps. The predicted softened halo core densities depend critically upon the velocity dependence of the annihilation cross section. We show that from galactic satellites to clusters of galaxies, the halos all have nearly the *same* core density: $\rho_{\rm core} \sim 0.02\,M_\odot/\mathrm{pc}^3 = 0.8\,\mathrm{GeV}/\,\mathrm{cm}^3$. This remarkable fact implicates s-wave annihilation. We discuss the constraints on models for annihilating CDM, which point to a candidate beyond those currently favored.

95.35.+d; 95.30.Cq; 98.80.Es; 98.80.Cq

Introduction. The idea that the large-scale structure developed by gravitational instability from initially small-amplitude, adiabatic and nearly scale-invariant fluctuations is compatible with a number of observables across a wide range of length scales (e.g., from the cosmic microwave background anisotropy to the Lyman-α forest). Essential to this compatibility is the existence of cold dark matter: matter which is non-baryonic, has only very weak interactions with photons and baryons, and (prior to gravitational collapse) is cold.

The greatest challenge to this otherwise successful scenario comes from the apparent discrepancy between predicted dark-matter halo-density profiles and those inferred from observations. Simulations with non-interacting cold dark matter lead to halo density profiles that are singular at the center [1], whereas observations indicate uniform density cores. In this *Letter* we explore the possibility that the dark matter today has a large cross-section for annihilation which results in preferential destruction in high-density regions, softening halo cores.

Detecting and determining the properties of the dark matter is a major goal of observational cosmology. If annihilations are indeed altering the properties of dark matter halos, then we have a new means of studying the dark matter. The interactions of the CDM particles determine both the magnitude and velocity dependence of the annihilation cross section. For example, for s-wave annihilation, $\sigma_A|v|$ is independent of velocity and for p-wave annihilation, $\sigma_A|v|$ is proportional to v^2. These two different dependences result in different scaling relations between core density and halo velocity dispersion, which can be tested by current observations.

As we show below, current data for high velocity dispersion systems such as clusters of galaxies to low velocity dispersion systems such as galactic satellites are well-fit with the same core density of about $1\,\mathrm{GeV}/\,\mathrm{cm}^3$ ($= 0.026\,M_\odot/\,\mathrm{pc}^3$) [2]. This striking fact favors s-wave annihilation with a cross-section $\sigma v \sim 10^{-28}(m/\,\mathrm{GeV})\,\mathrm{cm}^2$, although future improvements in both the data and predictions will be necessary before such a statement can be made with confidence. As we shall discuss, the cosmological and astrophysical constraints on annihilating CDM point to a candidate beyond those currently favored (e.g., axion, neutralino).

Halos of Annihilating Dark Matter. Numerical simulations of structure formation in the CDM scenario show that the dark matter halos which form with a wide range of masses are all well-fit with the so-called NFW [1] form for the density profile. This form has $\rho \propto r^{-3}$ at large r and a cuspy inner region with $\rho \propto r^{-\alpha}$ with $\alpha = 1$. More recent higher resolution simulations predict cusps that are even stronger, with $\alpha \simeq 1.5$. Nevertheless, in most of what follows, we use the NFW theory for simplicity.

To be precise, the NFW profile is

$$\rho(r = xr_s) = \rho_s x^{-1}(1+x)^{-2}, \tag{1}$$

where the value of ρ_s is determined by the mean density of the Universe at the time the halo collapsed. In CDM theory, small objects collapse first, followed later by larger ones. Thus, there is an inverse relationship between ρ_s and halo size. In Fig. 1 we show this scaling relation with halo size represented by velocity dispersion for the halo, estimated as $\sigma_{\rm vir} = \sqrt{GM_{\rm vir}/2r_{\rm vir}}$. (The virial radius, $r_{\rm vir}$, is defined such that the mean density inside the $r_{\rm vir}$ sphere is 200 times the present mean density of the Universe, and $M_{\rm vir}$ is the mass contained within $r_{\rm vir}$ [1].)

Annihilations will alter the halo profiles near the core where the density of the dark matter particles is the highest. The annihilation rate (per particle) $\Gamma = n\langle\sigma|v|\rangle$ depends on the velocity dispersion. We parameterize the velocity dependence as $\Gamma = (\rho/m)\sigma_A v^n$ ($n = 0$ for s-wave; $n = 2$ for p-wave), where v is the velocity dispersion and m is the CDM particle mass.

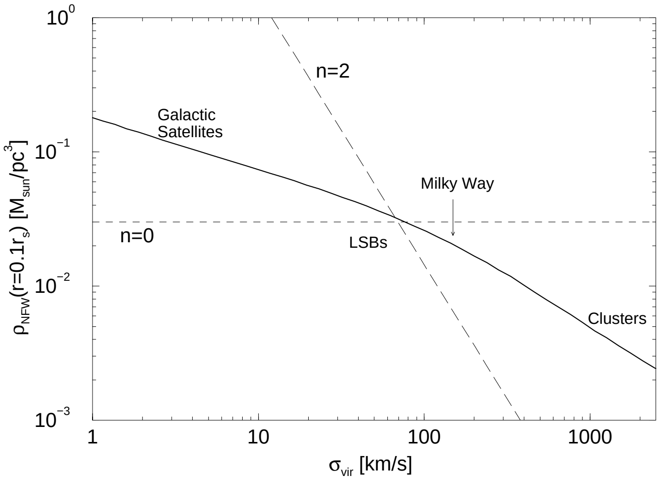

FIG. 1. The halo density at $r = 0.1r_s$ (where the cusp problem becomes prominent) in structures of different size according to NFW [1]. The objects are characterized by their virial velocity dispersion as indicated. Annihilation lines, normalized to LSB galaxies, are shown for the cases $n = 0$ and 2. Above the line, annihilations are very important (at $r = 0.1r_s$) and below the line they are unimportant. For $n = 0$, LSB and smaller objects have their cores softened significantly, while clusters do not, consistent with observations. For $n = 2$, clusters would be adversely affected.

Fig. 1 also shows in a qualitative way how annihilations affect the core structure of different objects. The annihilation lines drawn show whether or not annihilations are important at a NFW halo radius of $0.1r_s$ in different kinds of objects. (Note, since halo densities diverge, for any object annihilations become significant deep enough into the core). The annihilation lines are normalized to soften the cores of low surface-brightness (LSB) spiral galaxies. Because of how annihilations scale with velocity, for n = 0 clusters remain unaffected at $r \gtrsim 0.1r_s$, while the cores of LSBs and smaller objects are dramatically softened. For n = 2, the opposite is true, which contradicts observations that indicate the NFW profile works well for clusters. We expect any $n > 1$ to be inconsistent with observations. The case of $n = 0.5$ is interesting since the annihilation line runs parallel to the structure line (for $\sigma_{\rm vir} \lesssim 100$ km/s), implying that all the systems will be smoothed off at the same value of r/r_s.

Model Building Constraints. For the annihilations to be effective in galaxy cores today, the annihilation rate must satisfy the (approximate) constraint:

$$\Gamma \sim (\rho/\rho_{\rm LSB})\,(v/v_{\rm LSB})^n\,H_0\,, \tag{2}$$

where the subscript LSB denotes the appropriate values for a typical LSB and $H_0 = 100h$ km s^{-1} is the present expansion rate of the Universe. Outside collapsed objects today, the density of CDM is much lower and annihilations will be unimportant for $n \geq 0$. The early Universe is another matter as densities were much higher, $\rho \propto T^3$, where T is the cosmic background radiation temperature.

The figure of merit for the effectiveness of annihilations in the early Universe is measured by annihilation rate divided by the expansion rate: when $\Gamma/H > 1$ annihilations are effective (and vice versa). Assuming that the velocity dispersion of the CDM particles can be characterized by the background radiation temperature and normalizing the cross section to the desired value today, the temperature dependence of Γ/H is

$$\frac{\Gamma}{H} \sim 10^9 \left(\frac{T}{\rm GeV}\right)\left(\frac{T}{10^{-3}m}\right)^n \sqrt{\frac{T}{T+T_{\rm eq}}}, \tag{3}$$

where $T_{\rm eq} \sim 1\,{\rm eV}$ is the temperature at matter – radiation equality. There are three important things to note: (1) the large coefficient in front of this expression – annihilations in the early Universe are a significant consideration; (2) for $n = -1$, the effectiveness of annihilations is epoch independent and disastrous; and (3) for $n > -1$ annihilations were more important in the past.

Observational data suggest that if halos are made of annihilating CDM particles, their annihilation cross section is characterized by $n \lesssim 1$. Thus we will focus on $n > -1$, where the danger of annihilations is in the past: $\Gamma/H > 1$ for

$$T > T_A \sim 10^{-3(3+n)/(1+n)}\,{\rm GeV}\,(m/{\rm GeV})^{n/(1+n)}\,, \tag{4}$$

or $1\,{\rm eV}$ for $n = 0$. To ensure that early annihilations do not reduce CDM particles to negligible numbers, they must be protected against annihilation in the early Universe. We suggest two mechanisms; doubtless, there are other possibilities.

First, CDM particles could be produced late ($T < T_A$) by the decays of another massive particle. Although this requires a long lifetime, $\tau > t(T_A) \sim 10^5$ yrs, the lifetime does not have to be fine-tuned since $t(T_A) \ll t_0$.

The second way of avoiding the early-Universe annihilation catastrophe is to make the mass of the annihilation product (or CDM particle) dynamical. For example, a phase transition which takes place at $T < T_A$ could change annihilation from being kinematically impossible to possible if the mass of the annihilation product dropped below threshold after the phase transition (or if the mass of the CDM particle rose above threshold). A variation on this theme is coupling the annihilation produced particle to a scalar field, ϕ, with $\langle\phi\rangle \neq 0$. As $\langle\phi\rangle$ decreases, either quickly to zero as a result of a symmetry-restoring phase transition, or slowly as $\langle\phi\rangle$ rolls to the minimum of its potential, the product particle's mass may drop below threshold, opening up the new annihilation channel, at $T < T_A$.

Finally, the CDM annihilation products must not include photons because their γ-ray flux would far exceed observational limits. For example for 1 GeV CDM particles, the flux would be around $10^5\,\mathrm{cm}^{-2}\mathrm{sr}^{-1}\mathrm{s}^{-1}$, some ten orders of magnitude above the observed diffuse γ-ray flux at 1 GeV.

Observational Constraints. We henceforth restrict ourselves to $n = 0$. We solve for the annihilation-modified profile ignoring any gravitational back-reaction, and taking the initial profile to be NFW. The contribution of annihilations to the evolution of the density is given by

$$d\left[\rho/\rho_A\right]/dt = -\left[\rho/\rho_A\right]^2 t_0^{-1}\,, \tag{5}$$

where $\rho_A \equiv m/(\sigma_A t_0)$ and t_0 is the age of the Universe today. The resulting density profile today is

$$\rho(r) = \rho_s\left[x(1+x)^2 + \delta_A\right]^{-1}\,, \tag{6}$$

where $x \equiv r/r_s$, $\delta_A \equiv (1-t_c/t_0)\rho_s/\rho_A$, and t_c is the time when the collapse of the structure under consideration occurred. We will neglect t_c henceforth, but bear in mind that annihilations predict a slightly higher core density for younger (more massive) objects.

The appearance of a constant density core ($\rho \approx \rho_A$) is readily apparent. We define the core size, r_A, such that $\rho(r_A) = \rho_A/2$: for $\delta_A \ll 1$, $r_A = r_s\delta_A$. Two important points should be noted – one, the core size depends on the mass of the halo, and two, the universal value for the core density, *independent* of the mass of the collapsed structure.

We now turn to the observable constraints on annihilating CDM. As can be seen in Fig. 2, smaller systems depart from NFW at larger values of r/r_s. So we first turn to the galactic satellites in the Milky Way group [5], of which there are 11 known.

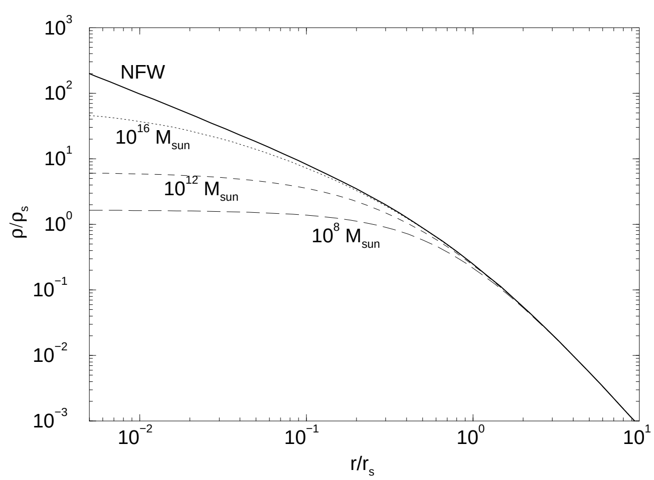

FIG. 2. Annihilation-modified density profiles. The curves are labeled by the virial mass of the halo: $10^{16}\,M_\odot$ (cluster), $10^{12}\,M_\odot$ (galaxy), and $10^8\,M_\odot$ (galactic satellite). The solid curve is the unmodified NFW profile. While the core densities for different systems are the same in physical units, they differ in units of ρ_s.

For a $10^8\,M_\odot$ galactic satellite, the core radius produced by annihilations is about 1 kpc, which is about the same as the cut-off radius induced by tidal forces. Hence these systems should have flat profiles, a prediction which seems to be consistent with observations. Most of the galactic satellites have large velocity dispersions ($\sim$ 10 km/s) for their stellar content, which suggests that they are CDM dominated [6]. If so, their internal velocity dispersions indicate that $\rho_A = \mathcal{O}(1\,\mathrm{GeV/\,cm^3})$ [9].

We also looked at dwarf spiral galaxies and LSBs. One must use these with caution since van den Bosch *et al* [10] have recently claimed that most of the H I rotation curve data do not have sufficient spatial resolution to put meaningful constraints on the halo cusps. They do identify three nearby galaxies which have sufficient spatial resolution – NGC 247, DDO 154 and NGC 3109. The two dwarfs, DDO 154 and NGC 3109, were shown by Moore [11] to be inconsistent with standard CDM predictions. van den Bosch *et al* [10] find that $0.55 < \alpha < 1.26$ for the LSB (NGC 247), and $\alpha < 0.5$ for the two dwarfs, at the 99.73% confidence level. At face value, this is evidence that only low-mass systems have soft cores.

These results can also be explained with the annihilation scenario. Since r_A/r_s is smaller for the higher mass systems, the core is less evident. We fit to the three galaxies identified above with the halo profile in Eq. 6, a thin stellar disk, the observed gas and including the effect of finite resolution. We find that $\rho_A = 0.02\,M_\odot/\,\mathrm{pc}^3$ results in a good fit to all 3 galaxies (see Fig. 3). Mass to light ratios (in solar units) of 5.5 for NGC 247, and 1.4 for DDO 154 were obtained. In all three cases, the outer parts of the halo (determined by ρ_s and r_s) are consistent with NFW theory.

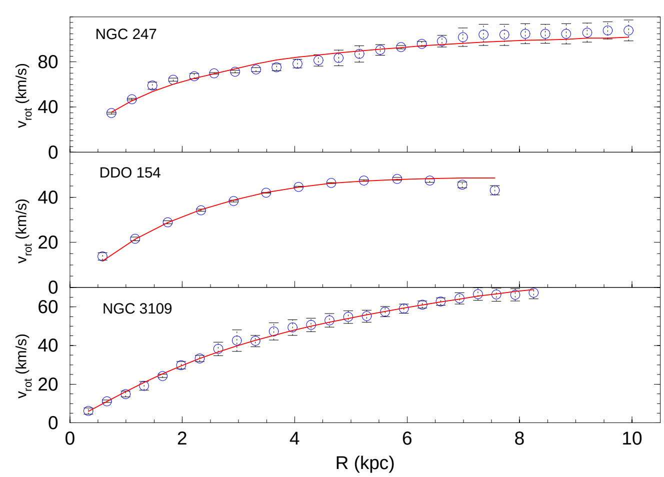

FIG. 3. Rotation curve fits with $\rho_A = 0.02\,M_\odot/\,\mathrm{pc}^3$.

For evidence on the largest scales, we turn to observations of strong gravitational lensing of background galaxies by clusters. Tyson *et al* [12] model the mass distribution in the cluster CL 0024+1654, which produces multiple distorted images of a background galaxy, and find evidence for a compact soft core (of about $35h^{-1}$ kpc) in the projected density. A similar value for the core radius was inferred earlier by Smail *et al* [12] for CL 0024 and other clusters. X-ray studies (Bohringer *et al* [12]) of CL 0024 are also in agreement with the above results.

We find that the annihilation-modified NFW form in Eq. 6 with $\rho_A = 0.07h^2\, M_\odot/\,\mathrm{pc}^3$ (consistent with LSB rotation curve fits), $r_s = 660h^{-1}$ kpc and $\delta_A = 0.02$ result in a surface density consistent with its reconstruction by Tyson *et al* [12]. The implied CDM mass within the arc radius (of $107\,h^{-1}$ kpc) is in agreement with the quoted value of about $1.66 \times 10^{14}\,h^{-1}\,M_\odot$ for the total mass within the arc radius [13].

Discussion. The invariance of the core density over 8 orders-of-magnitude in mass is remarkable [2]. It is intriguing that the s-wave CDM annihilation scenario predicts just that. This implies an upper limit to the core density of old objects of $\rho_A \equiv m/(\sigma_A t_0)$. A universal core density of $1\,\mathrm{GeV}/\,\mathrm{cm}^3$, which is consistent with s-wave annihilation and all observations that we are aware of, implies $\langle\sigma|v|\rangle = 0.8 \times 10^{-28}(m/\,\mathrm{GeV})\,\mathrm{cm}^2$.

In addition to the cusp crisis, simulations of non-interacting CDM also predict a much larger number of sub-halos for a galactic size halo than the observed number of galactic satellites [14]. Certainly, the s-wave annihilation scenario has a dramatic effect on the smallest halos, and this could contribute to their destruction. However, further study is required to test this hypothesis.

Spergel and Steinhardt [15] have proposed a different particle physics solution in which the large self-scattering cross-section of CDM particles is supposed to soften halo cores. However, it has since been shown that self-scatterings lead to cores which are more spherical than observed [16], and simulations suggest self-scatterings steepen the central cusp of the halo [17].

The requirements on a model for annihilating CDM are stringent, but by no means impossible [18]. They point to a particle beyond those currently being considered, and therefore, to new physics. While it is possible that the solution to the CDM cusp problem will involve the interpretation of the observations or less exotic astrophysics, it is appealing to think that the properties of halo cores may teach us about the fundamental properties of the CDM particle.

ACKNOWLEDGMENTS

We gratefully acknowledge support by the DOE, NASA and NSF at The University of Chicago, and by the DOE and NASA grant NAG 5-7092 at Fermilab. We thank M. Valluri for useful discussions and pointers throughout this project. We also thank S. Carroll, D. Eisenstein, and L. Widrow for useful discussions.

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